CONTEMPORARY RESEARCH IN INDIA
ISSN 2231 - 2137

# THENECESSARY AND SUFFICIENT CONDITIONS FOR CONFORMALLY SYMMETRIC SPACE-TIME

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| Received: 30/08/2018 | Edited: 06/09/2018 | Accepted: 13/09/2018 |
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***Abstract:*** *The space-time of infinitely conducting relativistic charged fluid is studied under the caption of conformally symmetric spaces. The necessary and sufficient conditions are found for conformally symmetric space-time to be symmetric space time.*
***Keywords:*** *Relativisticfluid, Ricci tensor, conformal wely tensor.*

**Introduction:**

We consider the stress-energy tensor for relativistic Magnetohydrodynamics (Shaha, 1974) in the form

$$T_{ab} = (\rho + m)U_a U_b + (P_1 - m)V_a V_b + (P_2 + m)W_a W_b + (P_3 + m)N_a N_b \quad \ldots (1.1)$$

Where the unitary orthonormal tetrad (U, V, W, N) is chosen such that

$$H^a = |H|V^a$$

$$H^a H_a = -H^2, \quad \ldots (1.2)$$

$$m = \frac{1}{2}\mu H^2$$

And $P_1, P_2, P_3$ are hydrostatic pressures acting along the directions V, W and N respectively. Moreover, $\rho$ is energy density.

Also $U^a U_a = -V_a V^a = -W^a W_a = -N_a N^a = 1$

$$U^a V_a = U^a W_a = U^a N_a = V^a W_a = V^a N_a = W^a N_a = 0 \quad \ldots (1.3)$$

**Remark**:The fluid described by (1.1) is designated here after as Magnetofluid. The field equations for the system consist of

**(a) Einstein's Field Equations**

In terms of the symmetric Ricci tensor $R_{ab}$Ricci scalar R and symmetric stress-energy tensor $T_{ab}$, The famous Einstein's field equation are written in the form

$$R_{ab} - \frac{1}{2}R g_{ab} = -K T_{ab} \ldots (1.4)$$

With $T_{ab}$ given by (1.1)

**(b) Maxwell's Equations**

The condition of infinite conductivity demands that the only valid set of Maxwell's equations is in the form

$$|\bar{H}|(V^a U^b - U^a V^b)_{;b} = |\bar{H}|(U^a V^b - U^b V^a)_{;b} \quad \ldots (1.5)$$

The Ricci rotation coefficient with respect to above terad are defined as

$$\gamma\alpha\beta\delta = \lambda_{\alpha a;} b \lambda^{\alpha}{}_{\beta} \lambda^{b}{}_{\delta} \quad \ldots (1.6)$$

Where $\alpha, \beta, \delta$ are terad indices, a b are tensor indices and $\lambda^a{}_\alpha$are terad vectors.

Hence $\underset{\alpha\beta\delta}{\gamma} = \underset{\beta\alpha\delta}{-\gamma}$ ... (1.7)

The equation (1.1) and (1.4) will provide the expression for Ricci tensor in the form

$$R_{ab} = -K[(\rho + m)U_a U_b + (P_1 - m)V_a V_b + (P_2 + m)W_a W_b + (P_3 + m)N_a N_b - \frac{1}{2}(\rho - P_1 - P_2 - P_3)g_{ab}]$$

... (1.8)

The consequences of Maxwell's equations (1.5) are

$\underset{421}{\gamma} = \underset{124}{\gamma}$, ... (1.9)

$\underset{431}{\gamma} = \underset{134}{\gamma}$, ... (1.10)

$|\bar{H}|\left[\underset{122}{\gamma} + \underset{133}{\gamma}\right] = \left|\dot{\bar{H}}\right|, aV^a$ ... (1.11)

$|\bar{H}|\left[\underset{422}{\gamma} + \underset{433}{\gamma}\right] = \left|\dot{\bar{H}}\right|, aU^a$ ... (1.12)

According to Chaki and Gupta, 1963 the conformally symmetric space-time is defined through the condition that the Wely Tensor is covariantly constant.

$C_{abcd;e} = 0$ ... (1.13)

Further, the symmetric space-time has the defining condition [Yano, 1957]

$R_{abcd;e} = 0$ ... (1.14)

**Theorem:** The conformally symmetric space-time associated with the Magnetofluid characterized by the stress-energy tensor having vanishing sum of eigen values is symmetric if and only if

$$\rho; c = P_{1;c} = P_{2;c} = P_{3;c} = H^2{}_{;c} = 0$$

And

$$\underset{\alpha 44}{\gamma} = 0 = \underset{4\alpha\beta}{\gamma}, \alpha, \beta = 1,2,3,$$

$$\underset{133}{\gamma} = \underset{233}{\gamma} = \underset{211}{\gamma} = \underset{322}{\gamma} = 0.$$

$$\underset{134}{\gamma} = \underset{123}{\gamma} = \underset{132}{\gamma} = \underset{234}{\gamma} = 0.$$

**Proof:** We have the Weyl tensor expression

$C_{abcd} = R_{abcd} - \frac{1}{2}(g_{ac}R_{bd} + g_{bd}R_{ac} - g_{bc}R_{ad} - g_{ad}R_{bc}) - \frac{1}{6}(g_{ac}g_{bd} - g_{ad}g_{bc})R$ ... (1.15)

$C_{abcd;e} = R_{abcd;e} - \frac{1}{2}\left(g_{ac}R_{bd;e} + g_{bd}R_{ac;e} - g_{bc}R_{ad;e} - g_{ad}R_{bc;e}\right) - \frac{1}{6}R_{;e}(g_{ac}g_{bd} - g_{ad}g_{bc})$ ... (1.16)

Now if the space-time is symmetric then

$R_{abcd;c} = 0,$ ... (1.17)

Which implies that

$R_{ab;c} = 0,$ ... (1.18)

And

$R_{;c} = 0,$ ... (1.19)

Hence (1.16) gives

$C_{abcd;e} = 0,$ ... (1.20)

Consequently, the space-time is conformally symmetric. Further, to prove the converse part, we observe from (1.16) that the conformally symmetric space-time $[C_{abcd;e} = 0]$ becomes symmetric $[R_{abcd;c} = 0]$ if and only if

$R_{ab;c} = 0,$ ... (1.21)

Hence we find the necessary and sufficient conditions for this. But we have the expression for Ricci tensor corresponding to the relativistic Magnetofluid,

$R_{ab} = -K[AU_aU_b + BV_aV_b + CW_aW_b + DN_aN_b]$ ... (1.22)

Where

$$A = m + \frac{1}{2}(\rho + P_1 + P_2 + P_3),$$
$$B = \frac{1}{2}(P_1 - P_2 - P_3 + \rho) - m,$$
$$C = m + \frac{1}{2}(\rho - P_1 + P_2 - P_3),$$
$$D = m + \frac{1}{2}(\rho - P_1 - P_2 + P_3).$$

Hence (1.21) yields

$A_{;c}U_aU_b + AU_{a;c}U_b + AU_aU_{b;c} + B_{;c}V_aV_b + BV_{a;c}V_b + BV_aV_{b;c} + C_{;c}W_aW_b + CW_{a;c}W_b + CW_aW_{b;c} + D_{;c}N_aN_b + DN_{a;c}N_b + DN_aN_{b;c} = 0$ ... (1.23)

If we contract this equation within $U^aU^b, V^aV^b, W^aW^b and\ N^aN^b$ then we get

$A_{;c} = B_{;c} = C_{;c} = D_{;c} = 0$ ... (1.24)

These in term imply

$\rho_{;c} = P_{2;a} = P_{3;a} = -P_{1;a}$ ... (1.25)

Now the sum of eigen values of stress-energy tensor (1.1) vanishes then it gives with the usage of (1.25)

$P_{1;a} = M_{;1}$ ... (1.26)

Thus, the conditions (1.25) and (1.26) finally produce the result

$\rho_{;c} = P_{a;c} = P_{2;c}P_{3;c} = H^2{}_{;c} = 0.$ ... (1.27)

Again the further contradiction of (1.23) provide the results

$$\underset{\alpha 44}{\gamma} = 0 = \underset{4\alpha\beta}{\gamma} \quad \alpha\beta = 1,2,3$$

$\underset{133}{\gamma} = \underset{233}{\gamma} = \underset{211}{\gamma} = \underset{322}{\gamma} = 0,$ ... (1.28)

$$\underset{134}{\gamma} = \underset{123}{\gamma} = \underset{132}{\gamma} = \underset{234}{\gamma} = 0,$$

Thus the conditions (1.27) and (1.28) become the necessary and sufficient conditions for the validity of the theorem.

**Conclusion:**

(1) The conditions (1.27) show that the mass energy-density and hydrostatic pressures in all directions are covariantly constant. Moreover, the magnitude of the magnetic field is also preserved.

(2) Ti reveals from (1.28) that the streamlines of the Magnetofluid are geodesic in nature and are expansion-free.

Remark It is provided by Sharma, 1988 that for conformal symmetric space-time admitting a proper conformal vector field, is either of type O or N.

Note (1) A new space called C space is characterized by Asgekar and Date, 1978 as $J^*_{abc} = C^d{}_{abc;d} = 0$.But this condition is supplied by the symmetric space - time condition.

Hence, We can state that C space forms a particular case of conformally symmetric space-time.

Note (2) The gravitational tidal force due to Ellis, 1967 is defined as $[C_{abcd}\, u^c u^d]$. It is proved by Asgekar and Date, 1978 that the tidal force is due to magnetic field only under the condition of C space. The same result is valid for symmetric space – time of Magnetofluid.

**2. Concluding Remarks**

The powerful technique of Ricci rotation coefficients is employed to examine the effect of geometrical symmetries on the conservation on dynamic quantities. Mainly the inter-relationship between symmetric and conformally symmetric space – time is observed through the theorem. The necessary and sufficient conditions are developed in this case.

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