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PERFECT MAGNETOFLUID ADMITS INERTIAL REFERENCE FRAME

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Abstract: *It provides that if the space time of perfect Magnetofluid admits inertial reference frame then:*
(i) The matter energy density is preserved along the flow lines.
(ii) The isotropic pressure is conserved along the magnetic lines.
(iii) The magnetic lines of perfect Magnetofluid admitting IRF are expansion free if and only if the magnitude of the magnetic field is preserved along these lines.

Keywords: *Inertial Reference Frame (IRF), perfect Magnetofluid, Expansion free.*

1. Introduction:

A Surve of literature exposes the various types of tetrads used in the general theory of relativity which can be categorized in the following types:

(1) One time like and 3 space like vector fields due to Eisenhart, 1960.
(2) One null and three space like due to Synge, 1972.
(3) Two null and two space like vectors due to Hall, 1977.
(4) Two real and two complex null vectors due to Newman-Penrose, 1962.

This paper deals with first type of tetrad fields. Thus the most advantages formalism of orthonormal tetrad fields possessing sixteen independent components and exhibiting a geometry which is more general than Riemannian geometry is described through a set of tetrad vectors $\lambda^{a}_{(m)}$ (the bracketed letters denote tetrad suffixes and non- bracketed letters denote tensor indices) satisfying the following relations:

$$\left.\begin{array}{l} \lambda^{a}_{(m)}\lambda^{(m)}_{b} = \delta^{a}_{b} \; ; \; \lambda^{a}_{(m)}\lambda^{(n)}_{a} = \delta^{n}_{m} \\ \lambda^{a}_{(m)}\lambda_{a(n)} = g_{mn} \; ; \; \lambda_{(m)a}\lambda^{(m)}_{b} = g_{ab} \end{array}\right\} \quad \ldots (1.1)$$

So also the associated Ricci rotation coefficients are given by

$$\gamma_{\alpha\beta\delta} = \lambda_{(\alpha)a;b}\lambda^{a}_{(\beta)}\lambda^{b}_{(\delta)} \quad \ldots (1.2)$$

The tetrad vector fields are taken as

$$\lambda^{a}_{(m)} \equiv (V^{a}, W^{a}, N^{a}, U^{a}) \quad \ldots (1.3)$$

Subject to conditions

$$U_{a}U^{a} = -V_{a}V^{a} = -W_{a}W^{a} = -N_{a}N^{a} = 1 \quad \ldots (1.4)$$

And

$$U_aV^a = U_aW^a = U_aN^a = V_aW^a = V_aN^a = W_aN^a = 0 \qquad \dots (1.5)$$

Thus the choice for tetrad field vector is such that one vector $\overline{U}$ is time like and other three unite vectors V^a, W^a, N^a are space like.

Remark: It follows from the defining expression (1.2)

$$\underset{\alpha\beta\delta}{\gamma} = -\underset{\beta\alpha\delta}{\gamma}.$$

2. Anisotropic Magnetofluid Scheme

In 1955, Lichnerowicz has suggested a definite material scheme in relativistic hydrodynamics exposed through the stress energy tensor.

$$\underset{m}{T_{ab}} = \rho U_aU_b + P_1V_aV_b + P_2W_aW_b + P_3N_aN_b. \qquad \dots (2.1)$$

Here, ρ is the matter energy density and P_1, P_2, P_3 $(P_1 \neq P_2 \neq P_3)$ are the anisotropic pressures acting along the 3-orthonormal space directions V^a, W^a, N^a . The tetrad field vectors(V^a, W^a, N^a, U^a) satisfy the relations (1.4) and (1.5).

Note: If $P_1 = P_2 = P_3 = P$ the equation (2.1) becomes

$$T_{ab} = \rho U_aU_b + P(V_aV_b + W_aW_b + N_aN_b) \qquad \dots (2.2)$$

i.e. $T_{ab} = (\rho + P)U_aU_b + Pg_{ab}$ $\qquad \dots (2.3)$

[Greenberg, 1970b and Shaha 1974].

This is well – known form of relativistic perfect fluid. Hence by looking at the nature that the three spatial pressures (P_1, P_2, P_3)acts along the three different directions $(\overline{V}, \overline{W}, \overline{N})$. We suppose that the stress energy tensor (2.1) describes relativistic anisotropic fluid.

The scheme of relativistic magneto hydrodynamics as designed by Lichnerowicz (1967) consisting of a relativistic charged perfect fluid with infinite conductivity and constant magnetic permeability. According to the formation of this scheme, the condition of infinite conductivity with the principle of conservation of current demands the condition of zero electric field. Thus the electromagnetic field in this situation gets reduced to magnetic field only with respect to the velocity of the considered fluid. Consequently the stress energy tensor for electromagnetic field under the assumption of infinite conductivity and constant magnetic permeability (μ) takes the form.

$$\underset{em}{T_{ab}} = \mu\left[\left(\frac{1}{2}g_{ab} - U_aU_b\right) - h_ch^c - h_ah_b\right].. \qquad .. (2.4)$$

Here, the magnetic field vector h_a is space like and satisfies the properties

$$U^ah_a = 0. h^ah_a = -H^2 \qquad \dots (2.5)$$

It follows from these results

$$h^a{}_{;b}h_a = -h^a{}_{;b}U_a \qquad \dots (2.6)$$

And

$$h^a{}_{;b}h_a = -\frac{1}{2}H^2{}_{;b} \qquad \dots (2.7)$$

The magnetic field exhibited by the tensor (2.4) is subject to satisfy the only valid set of Maxwell's equation given by

$$(U^ah^b - U^ah^a)_{;b} = 0 \qquad \dots (2.8)$$

Amalgamation of matter field andElectromagnetic Field

A more general anisotropic Magnetofluid scheme (Shaha, 1974) can be formulated by the mixture of the two stress energy tensors given by equation (2.1) and (2.4) in the form

$$T_{ab} = \underset{m}{T_{ab}} + \underset{em}{T_{ab}} \quad \dots (2.9)$$

i.e. $$T_{ab} = \rho U_a U_b + P_1 V_a V_b + P_2 W_a W_b + P_3 N_a N_b + \mu \left[\left(\frac{1}{2} g_{ab} - U_a U_b \right) - h_c h^c - h_a h_b \right]. \quad \dots (2.10)$$

If we choose the magnetic field in the direction of the tetrad vector V^a, that is

$$V^2 = \frac{h^a}{H} \equiv H^2 \quad \dots (2.11)$$

Then the form of equation (2.10) changes to

$$T_{ab} = \left(\rho + \frac{1}{2}\mu H^2\right) U_a U_b + \left(P_1 - \frac{1}{2}\mu H^2\right) V_a V_b + \left(P_2 + \frac{1}{2}\mu H^2\right) W_a W_b + \left(P_3 + \frac{1}{2}\mu H^2\right) N_a N_b \quad \dots (2.12)$$

i.e. $T_{ab} = AU_a U_b + BV_a V_b + CW_a W_b + DN_a N_b \dots$ (2.13)

with the values

$$A = \left(\rho + \frac{1}{2}\mu H^2\right) \quad \dots (2.14)$$

$$B = \left(P_1 - \frac{1}{2}\mu H^2\right) \quad \dots (2.15)$$

$$C = \left(P_2 + \frac{1}{2}\mu H^2\right) \quad \dots (2.16)$$

$$D = \left(P_3 + \frac{1}{2}\mu H^2\right) \quad \dots (2.17)$$

Thus the anisotropic Magnetofluid scheme is described by the stress energy tensor (2.13) in terms of orthonormal tetrad the vector fields (V^a, W^a, N^a, U^a) with magnetic field acting along V^a. The magnetic field part in (2.13) is subject to satisfy Maxwell's equation given by (2.8). These Maxwell's equation with the choice of

$$h^a = HV^a \quad \dots (2.17a)$$

Take the form

$$(U^a HV^b - U^b HV^a)_{;b} = 0 \quad \dots (2.18)$$

i.e. $$H_{;b}(U^a V^b - U^b V^a) + H(U^a HV^b - U^b HV^a)_{;b} = 0 \quad \dots (2.19)$$

Remark (1): This scheme described by the stress energy tensor (2.1) is known to be compatible with class one space times (Pandey and Gupta, 1970).

Remark (2): The Lichnerowicz (1967) for thermodynamical perfect fluid with infinite electrical conductivity and constant magnetic permeability can be recovered from the expression (2.13) by putting $P_1 = P_2 P_3 = p$. The form of this stress energy tensor is given by

$$\bar{T}_{ab} = (\rho + p + \mu H^2) U_a U_b - \left(p + \frac{1}{2}\mu H^2\right) g_{ab} - \mu H^2 H_a H_b. \quad \dots (2.20)$$

This we describe as the perfect Magnetofluid scheme.

3. Field equations for anisotropic Magnetofluid scheme

The main equations governing the behavior of the motion of relativistically moving particles in the space time of anisotropic Magnetofluid is characterized by two types of differential relations

1. Einstein's Field Equations,
2. Maxwell's Equations.

(1) Einstein's Field Equations:

The geometrical and dynamical structure of the space time manifold is described through the well-known Einstein's Field Equations. These are ten independent, non-linear differential equations of order two which establish a definite relation between Ricci tensor R_{ab}, Ricci scalar R and stress energy momentum tensor T_{ab} in the form

$$R_{ab} - \frac{1}{2} R g_{ab} = -KT_{ab}. \quad \dots (3.1)$$

Where

$$K = \frac{8\pi G}{C^4}$$

Is the coupling constant and G is Newtonian gravitational constant. Here the right hand side T_{ab} of the expression (3.1) will explore the dynamical features of the anisotropic Magnetofluid under consideration where-as the left hand side of (3.1) describes the geometrical features of the space time.

(2) Maxwell's Equations:

In accordance with the approximation of infinite conductivity we get the only set of Maxwell equations followed by the magnetic blade as suggested by Lichnerowicz (1967) are

$$[H(U^2H^b - U^bH^a)]_{;b} = 0 \quad \dots (3.2)$$

i.e. $U^aH^bH_{;b} - H_{;b}U^bH^a + HU^aH^b{}_{;b} + HH^bU^a{}_{;b} - HU^b{}_{;b}H^a - HU^bH^a{}_{;b} = 0 \quad \dots (3.3)$

If we contract suitably this equation with V^a, W^a, N^a, U^a then we get the following consequences of Maxwell equation (In terms of Ricci rotation coefficients)

$$H\left[\underset{122}{\gamma} + \underset{133}{\gamma}\right] = H_{;a}V^a \quad \dots (3.4)$$

$$H\left[\underset{422}{\gamma} + \underset{433}{\gamma}\right] = H_{;a}U^a \quad \dots (3.5)$$

$$\underset{421}{\gamma} = \underset{124}{\gamma} \quad \dots (3.6)$$

$$\underset{431}{\gamma} = \underset{134}{\gamma} \quad \dots (3.7)$$

4. The space time of perfect Magnetofluid

The space time is characterized by stress energy tensor (2.20)

$$T_{ab} = (\rho + p + \mu H^2)U_aU_b - \left(p + \tfrac{1}{2}\mu H^2\right)g_{ab} - \mu h_a h_b \quad \dots (4.1)$$

By making use of field equations (3.1) we find the value

$$R_{ab}U^aU^b = -\frac{K}{2}\left[T_{ab} - \frac{1}{2}Tg_{ab}\right]U^aU^b \quad \dots (4.2)$$

This expression with equation (4.1) generates the result.

$$R_{ab}U^aU^b = -\frac{K}{2}[\rho + 3p + 3\mu H^2] \equiv M, \text{(say)} \quad \dots (4.3)$$

Hence the result due to Ehlers and Kundt, 1962 and Ellis, 1971 in the form,

$$R_{ab}U^aU^b = \dot{\theta} + \frac{1}{3}\theta^2 + 2(\sigma^2 - w^2) - U^a{}_{;a} \quad \dots (4.4)$$

Get reduced to

$$-\frac{K}{2}[\rho + 3p + 3\mu H^2] = \dot{\theta} + \frac{1}{3}\theta^2 + 2(\sigma^2 - w^2) - U^a{}_{;a} \quad \dots (4.5)$$

In order to derive the equation of continuity and stream lines of the perfect Magnetofluid, we use the stress energy tensor (4.1) in local energy balance equations $T^{ab}{}_{;b} = 0$. Hence we get the continuity equation and fluid path lines equation in the form

$$\dot{\rho} + (\rho + p)\theta = 0, \dot{\rho} = \rho_{;b}u^b \quad \dots (4.6)$$

And

$$A\dot{U}^c = B'_{;b}h^{bc} + \mu(h^ah^b)_{;b}h^ca \quad \dots (4.7)$$

Where

$$A = \rho + p + \mu H^2, B = p + \frac{1}{2}\mu H^2 \quad \dots (4.8)$$

5. Inertial reference frame and the space time of perfect Magnetofluid

A concept of central meaning in, Newtonian physics in the inertial reference frame (IRF). It may be defined as such a frame, in which no inertial forces occur. Nevertheless, general relativity too needs the concept of reference frame, because the results of measurement always depend on the motion of the respective observer. Accordingly, reference frame represented by observer fields are necessary to establish a connection between the mathematical quantities (defined independently of reference frame) and the measured variables (dependent the observer). Uhlmann, 1960, Dehnen 1970 reference frames form an essential part of general relativity. By giving the physical definition of Initial Reference Frame (IRF), Audretsch (1971) has studied the properties of solutions of Einstein field equations admitting IRF. Accordingly a space time admitting IRF satisfy the following kinematical properties

$$\theta = -\left(\underset{411}{\gamma} + \underset{422}{\gamma} + \underset{433}{\gamma}\right) = 0 \quad \ldots (5.1)$$

$$W_{ab} = \tfrac{1}{2}\left(\underset{412}{\gamma} - \underset{421}{\gamma}\right)(V_a W_b - V_b W_a) + \tfrac{1}{2}\left(\underset{423}{\gamma} - \underset{432}{\gamma}\right)(W_a N_b - N_a W_b) + \tfrac{1}{2}\left(\underset{431}{\gamma} - \underset{413}{\gamma}\right)(N_a V_b - N_b V_a) = 0 \quad \ldots (5.2)$$

$$\sigma_{ab} = \left(\underset{411}{\gamma} + \tfrac{1}{3}\theta\right) V_a V_b + \left(\underset{422}{\gamma} + \tfrac{1}{3}\theta\right) W_a W_b + \left(\underset{433}{\gamma} + \tfrac{1}{3}\theta\right) N_a N_b + \tfrac{1}{2}\left(\underset{412}{\gamma} - \underset{421}{\gamma}\right)(V_a W_b + V_b W_a) + \tfrac{1}{2}\left(\underset{423}{\gamma} - \underset{432}{\gamma}\right)(W_a N_b + N_a W_b) + \tfrac{1}{2}\left(\underset{413}{\gamma} - \underset{431}{\gamma}\right)(N_a V_b + N_b V_a) = 0 \quad \ldots (5.3)$$

$$\dot{U}^a = -\left[\underset{414}{\gamma} V^a + \underset{424}{\gamma} W^a + \underset{434}{\gamma} N^a\right] = 0 \quad \ldots (5.4)$$

From the above conditions we write

$$\underset{4\alpha\beta}{\gamma} = -\underset{4\beta\alpha}{\gamma}, \alpha, \beta = 1,2,3, \alpha \neq \beta \quad \ldots (5.5)$$

And

$$\underset{411}{\gamma} = \underset{422}{\gamma} = \underset{433}{\gamma} = 0 \quad \text{[vide (5.1) – (5.2)]} \quad \ldots (5.6)$$

$$\underset{4\alpha\beta}{\gamma} = \underset{4\beta\alpha}{\gamma}, \alpha, \beta = 1,2,3 \quad \text{[vide (5.3)]} \quad \ldots (5.7)$$

$$\underset{\alpha 44}{\gamma} = 0 \quad \text{[vide (5.4)]} \quad \ldots (5.8)$$

Accordingly, the gradient of the flow vector becomes zero,

i.e. $U_{a;b} = 0.$ … (5.9)

This result can be dynamically interpreted as "3- momentum and energy of the freely moving test particle as measured by these inertial observers do not change with time".

We now, will investigate the properties of perfect Magnetofluid space time described by stress energy tensor (4.1) admitting IRF. The set of Maxwell equations (3.4) – (3.7) under the IRF conditions give the following results

$$h^b{}_{;b} = 0 \quad \ldots (5.10)$$

$$H^2{}_{;b} U^b = 0. \quad \ldots (5.11)$$

Hence we conclude that

(i) The magnetic lines are divergence free [vide equation (5.10)].

(ii) The magnitude of the magnetic field is conserved along the world line [vide equation(5.11)].

Theorem: If the perfect Magnetofluid space time admits IRF then

(i) The matter energy density is preserved along the flow lines,

(ii) The isotropic pressure is conserved along the magnetic lines.

Proof:

(i) If we use conditions (5.1) – (5.4) of IRF in the continuity equation (4.6) of the perfect Magnetofluid then we get

$$\rho_{;a}U^a = 0 \qquad \ldots (5.12)$$

(ii) Again the equation of stream lines for the perfect Magnetofluid (4.8) yields the result

$$\left(P + \frac{1}{2}\mu H^2\right)_{;c} h^{ac} + \mu)(h^c h^b)_{;b} h^a{}_c = 0 \qquad \ldots (5.13)$$

This with equation (5.10) generates

$$\left(p + \frac{1}{2}\mu H^2\right)_{;c} h^{ac} + \mu h^a{}_{;b} h^b = 0. \qquad \ldots (5.14)$$

If we transvect this equation by h_a then we get

$$P_{;c} h^c = 0 \qquad \ldots (5.15)$$

Thus equations (5.12) and (5.15) justify the validity of the theorem.

Remark: The equation (4.4) under the conditions of inertial reference frame reduces to

$$R_{ab} U^a U^b = -\dot{U}^a{}_{;a} \qquad \ldots (5.16)$$

This for perfect magnetofluid yields

$$\dot{U}^a{}_{;a} = \frac{\overline{K}}{2}(\rho + 3p + 3\mu H^2) \qquad \ldots (5.17)$$

This provides the expression for active gravitational mass-density for perfect magnetofluid under the condition of inertial reference frame (Ellis, 1971).

Note: The expression for the expansion parameter θ^* of magnetic field lines is given by

$$\theta^* = K^a{}_{;a} - U^a U^b K_{a;b} \qquad \ldots (5.18)$$

Where $K^a = \frac{h^a}{H}$

i.e. $\theta^* = \left(\frac{h^a}{H}\right)_{;a} - U^a U^b \left[\frac{h_a}{H}\right]_{;b}$

i.e. $\theta^* = \frac{h^a{}_{;a}H - h^a{}_{;a}}{h^2} - \left[\frac{h_{a;b}H - h_a H_{;b}}{H^2}\right] U^a U^b$

$\theta^* = -\frac{h^a H_{;a}}{H^2} + \frac{h_a H_{a;b} U^a U^b}{H^2}$ [vide (5.9) – (5.10)]

$$\theta^* = -\frac{h^a H_{;a}}{H^2} \text{[vide equation (5.11)]} \qquad \ldots (5.19)$$

Therefore,

$$\theta^* = 0 \Leftrightarrow H_{;a} h^a = 0 \qquad \ldots (5.20)$$

Hence we conclude that the magnetic lines of the perfect magnetofluid admitting Inertial Reference Frame are expansion free if and only if the magnitude of the magnetic field is preserved along these lines.

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