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GENERAL RELATIVISTIC MODELS FOR THE SPACE-TIME OF SPHERICALLY SYMMETRIC MAGNETOFLUID ADMITTING CONFORMAL MOTIONS

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Abstract: *Some General Relativistic models for the space-time of spherically symmetric Magnetofluid distribution (anisotropic in nature) admitting conformal motions are developed.*
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Introduction:

The phenomenon of local isotropy plays a vital role in the astrophysical studies of massive objects. However the present theoretical work on more relativistic stellar models suggests that the stellar matter may be anisotropic (Cosenza and Herrera et al., 1980). Anisotropy may be introduced by the existence of solid core in the presence of in electromagnetic field. We want to design suitable models for anisotropic matter in context of general relativity so that we can compare these with existing relativistic isotropic models. We have the anisotropic models given by Bicnell G.V. And Henriksen R.N. (1978) that show some properties of anisotropic spheres which are drastically differtent from the properties of isotropic spheres. We shall study the anisotropy caused by the presence of electromagnetic field.

We deal with a four dimensional space-time manifold with the metric given by
(in Schwarzschild co-ordinates)

$$ds^2 = A^2(r)dt^2 - B^2(r)dr^2 - r^2(d\Theta^2 + \sin^2\theta \, d\Theta^2) \qquad \ldots (1)$$

The field equations under investigation are

(a) Einstein's field equations given by

$$R_{ab} - \frac{1}{2}Rg_{ab} = -K \; T_{ab} \qquad \ldots (2)$$

Where T_{ab} is the stress-energy tensor for anisotropic Magnetofluid suggested by (Surve and Asgekar, 1988)

$$T_{ab} = (\bar{\varrho} + P)U_a U_b - \bar{p}g_{ab} + (P - \bar{p})H_a H_b, \qquad \ldots (3)$$

Here $\bar{\varrho}$ is the matter energy density,$\bar{p}$is the pressure along the direction normal to $\bar{u}$ and $\bar{H}$, μ is the constant magnetic permeability and P is the pressure along $\bar{H}$.The time-like flow vector $\bar{u}$and space-like magnetic field vector $\bar{H}$ satisfy the properties.

$$u^a H_a = 0, u^a u_a = -H^a H_a = 1 \qquad \ldots (4)$$

(b) Maxwell's equations

The only set of Maxwell's equations valid under the condition of infinite conductivity is (Lichnerowicz, 1967)

$$(u^a h^b - u^b h^a)_{;b} = 0, \qquad \ldots (5)$$

Where

$$h^a = hH^a \, , h^a h_a = -h^2 \, .$$

A space-time is said to admit conformal motions if and only if the gravitational potential g_{ab} satisfy the transformation equations

$$\underset{\bar{X}}{\pounds} g_{ab} = \emptyset g_{ab}, \ldots (6)$$

Where $\underset{\bar{X}}{\pounds}$ is the Lie derivative along the arbitrary vector $\bar{X}$ and ф is any arbitrary function of co-ordinates.

We have from equation (1)

$$g_{ab} = \begin{bmatrix} -B^2(r) & 0 & 0 & 0 \\ 0 & -r^2 & 0 & 0 \\ 0 & 0 & -r^2 sin^2\theta & 0 \\ 0 & 0 & 0 & A^2(r) \end{bmatrix} \quad \ldots (7)$$

For the choice of commoving frame the spherically symmetry yields

$$U^a = (U^0, 0,0,0), \quad H^a = (0, H^1, 0,0) \quad \ldots (8)$$

By utilizing the conditions

$$U^a U_a = -H^a H_a = 1,$$

We obtain the values of components

$$U^0 = \frac{1}{A(r)}, H^1 = \frac{1}{B(r)} \quad \ldots (9)$$

Hence the components of the energy momentum tensor (3) are

$$T_0^0 = \varrho, T_1^1 = -P, T_2^2 = T_3^3 = -\bar{P} \quad \ldots (10)$$

Now the field equation (2) corresponding to the Magnetofluid distribution can be written as

$$KP = \frac{1}{B^2}\left(\frac{2A\prime}{Ar} + \frac{1}{r^2}\right) - \frac{1}{r^2}, \quad \ldots (11)$$

$$K\bar{P} = \frac{1}{B^2}\left\{\frac{A''}{A} - \frac{A'B'}{AB} + \frac{1}{r}\left(\frac{A\prime}{A} - \frac{B\prime}{B}\right)\right\}, \quad \ldots (12)$$

$$K\bar{\varrho} = \frac{1}{B^2}\left(\frac{2B\prime}{Br} - \frac{1}{r^2}\right) + \frac{1}{r^2}, \ldots (13)$$

(Prime denotes differentiation with respect to r).

The function A and B are further restricted by the conditions (6), which implies in our case

$$A(r) = mr, \quad \ldots (14)$$

$$B(r) = \frac{2n}{\phi}, \quad \ldots (15)$$

Where m and n arbitrary constants of integration. Feeding (14) - (15) back into the field equations (11) – (13) We get

$$KP = \frac{3\phi^2}{4n^2}\left(\frac{1}{r^2}\right) - \frac{1}{r^2}, \quad \ldots (16)$$

$$K\bar{P} = \frac{\phi^2}{4n^2}\left\{\frac{2\phi\prime}{r\phi} + \frac{1}{r^2}\right\}, \quad \ldots (17)$$

$$K\bar{\varrho} = \frac{1}{r^2} - \frac{\phi^2}{4n^2}\left\{\frac{2\phi'}{r\phi} + \frac{1}{r^2}\right\}, \quad \ldots (18)$$

Further we have (Surve and Asgekar, 1988)

$$\left.\begin{array}{l} P = (p - ½\mu h^2), \\ P = (p + ½\mu h^2), \\ \bar{\varrho} = (\varrho + ½\mu h^2), \end{array}\right\} \quad \ldots (19)$$

Then by adding equation (16) and (17), we get

$$2KP = \left[\frac{\phi^2}{n^2r^2} - \frac{1}{r^2} + \frac{\phi\phi\prime}{2n^2r}\right]. \quad \ldots (20)$$

If we substract equation (17) and (16), then we get

$$2K\mu h^2 = -\frac{\phi^2}{n^2r^2} + \frac{2}{r^2} + \frac{\phi\phi'}{n^2r}. \quad \ldots (21)$$

The difference between (18) and (17) gives

$$2K\varrho = \frac{1}{r}\left[\frac{-3\phi\phi'}{n^2r} + \frac{1}{r}\right], \quad \ldots (22)$$

These equations (19) – (21) exhibit the values of pressure p, density ϱ and the magnitude of the magnetic field h^2, along the radial directions.

Note: By using Maxwell's equation (5) we get

$$(u^a h^b - u^b h^a)_{;b} = 0$$

i.e.$u^a; +b^{h^b} + u^a h^b; b - u^b h^a; b - u^b; b^{h^a} = 0.$ ... (23)

i.e.,$u^4; +b^{h^b} + u^4 h^b; b = 0 \; for \; a = 4.$

But we have

$$h^b; b = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial r}\left(\sqrt{-g}h^b\right),$$

i.e.$h^b; b = \frac{1}{r^2 ABsin\theta}\frac{\partial(r^2 ABsin\theta \; hH^b)}{\partial r},$

where$\sqrt{-g} = r^2 ABsin\theta$

Then this gives

$$h^b; b = (hH^b)' + \frac{2bH^b}{r} + hH^b\left(\frac{A'}{A} + \frac{B'}{B}\right),$$

Hence equation (22) implies

$$\frac{u^4;1(hH^1)}{u^4} + (hH^1)' + \frac{2hH^b}{r} + hH^1\left(\frac{A'}{A} + \frac{B'}{B}\right),$$

i.e.$\frac{(u^4)\prime}{u^4} + \frac{(hH^1)'}{hH^1} + \frac{2}{r} + \frac{A'}{A} + \frac{B'}{B} = 0.$

This yields a particular solution in the form

$hH^1 = \frac{C_1}{r^2 B}, C_1 \; constant$ of integration, ... (24)

i.e. $hH^1 = \frac{\bar{C}\emptyset}{r^2}, \bar{C} = \frac{C_1}{2n}$, vide (15),

i.e. $\frac{h}{B} = \frac{C\emptyset}{r^2}$, vide (9 and 15).

$\therefore h = \frac{C}{r^2} . C = 2n\bar{C}, \quad \because \emptyset \neq 0.$

i.e. $h^2 = \frac{C^2}{r^4}.$ ... (25)

Remarks: The magnitude of the magnetic field h^2, in case of static spherically symmetric space-time of Magnetofluid is given by Shaha (1984).

$h^2 = \frac{M}{r^4}$, M is constant.

Hence, our result (25) perfectly agrees with this result.

Theorem(1): For static Magnetofluid spheres admitting conformal motions $P = 0 \Longrightarrow \emptyset = C_1$ (constant).

Proof: The equation (16) with the condition P=0 produces

$$\frac{3\emptyset^2}{4n^2}\left(\frac{1}{r^2}\right) - \frac{1}{r^2} = 0.$$

This implies

$\emptyset = C_1.$ ... (26)

Remark: When we use (26) in equations (17) and (18) then we get

$$K\bar{P} = \frac{C^2}{4n^2r^2}, \quad \dots (27)$$

$$K\bar{\varrho} = \frac{1}{r^2} - \frac{C^2{}_1}{4n^2r^2}. \quad \dots (28)$$

Theorem (2): For static Magnetofluid spheres admitting conformal motions $\bar{P} = 0 \Longrightarrow \phi^2 = \frac{C_a}{r}$.

Proof: Divide equation (21) by 2 add in equation (20), so that by using $\bar{P} = 0$ supplies the result

$$\phi^2 + 2\phi\phi' r = 0$$

i.e. $\frac{\phi'}{\phi} = -\frac{1}{2r}$.

This differential equation has an exact solution

$$\phi = \sqrt{\frac{C_2}{r}},$$

i.e. $\phi^2 = \frac{C_2}{r}$, C_2 is constant of integration ... (29)

Remark: If we put $\phi^2 = \frac{C_2}{r}$ in equation (16) and (18) we get

$$KP = \frac{3C_2}{4n^2r^3} - \frac{1}{r^2} \quad \dots (30)$$

$$K\bar{\varrho} = \frac{1}{r^2} + \frac{2C^2{}_2}{4n^2r^{7/2}\sqrt{C}} - \frac{C^2}{4n^2r^2}. \quad \dots (31)$$

Theorem (3): For static Magnetofluid spheres admitting conformal motions $\bar{\varrho} = \bar{P} - P = 0 \Longrightarrow \phi = C_3$.

Proof: The condition $\bar{\varrho} = P$ gives with equation (16) and (18)

$$\frac{3\phi^2}{4n^2r^2} - \frac{1}{r^2} = \frac{1}{r^2} - \frac{\phi^2}{4n^2r^2}\left\{\frac{\phi^1}{r\phi} + \frac{1}{r^2}\right\}. \quad \dots (32)$$

This yields a differential equation

$\phi^2 + 2\phi\phi' r - 2n^2 = 0$.... (33)

Also the condition $\bar{\varrho} = \bar{P}$, with the equation (17) and (18) provides

$$\frac{\phi^2}{4n^2}\left\{\frac{2\phi^1}{r\phi} + \frac{1}{r^2}\right\} = \frac{1}{r^2} - \frac{\phi^2}{4n^2}\left\{\frac{2\phi^1}{r\phi} + \frac{1}{r^2}\right\}$$

i.e. $2n^2 - \phi\phi' r - \phi^2 = 0$.

... (34)

If we use (33) and (34) in the condition $\bar{\varrho} = P = \bar{P}$.

We get $\phi' = 0$.

This has an immediate integral

i.e. $\phi = C_3$, where C_3 is constant of integration

... (35)

Note: The conditions

$\underset{X}{\pounds} g_{ab} = \emptyset g_{ab}, \phi$ is constant

Describes the homothetic motion (Herrera et al., 1984). Hence we conclude that for the static Magnetofluid spheres, conformal motions into homothetic motions when

P=0 or $\bar{\varrho} = P = \bar{P}$. Vide (26) and (29).

Conclusions:

(1) The different values of ф[in equation (6)] lead to different class of solutions for anisotropic Magnetofluid space-time. Moreover we observe from (25) that, h^2 the magnitude of the magnetic field is a function of r. Hence we state that the space-time of anisotropic Magnetofluid admits conformal motions.

(2) For the static Magnetofluidspheres when P=0 or $\bar{\varrho} = P = \bar{P}$. Then conformal motion degenerate into homothetic motions.

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